

Yau College Math Competition 2020 Final Probability and Statistics

Overall Exam Problem Set (Saturday, October 24, 2020)

PROBLEM 1. Let $\{X_n\}$ be a sequence of independent, identically distributed random variables taking values in \mathbb{N} , the set of positive integers. Define

$$R_n = \text{Card}\{X_1, \dots, X_n\},$$

i.e., R_n is the number of distinct integers in the set $\{X_1, \dots, X_n\}$. Suppose that $\mathbb{E}[X_1] < \infty$. Prove that $R_n/\sqrt{n} \rightarrow 0$ in probability.

PROBLEM 2. Consider the mixture experiment whose components are E_1 and E_2 , taken with equal probabilities and each with the sample space $\Omega = \{0, 1, 2, \dots\}$. It is postulated that

- (1) the outcome of E_1 follows the Binomial model $\text{Binomial}(n, \theta)$ with the total number of trials $n > 0$ and the unknown probability of success parameter $\theta \in [0, 1]$, and
- (2) the outcome of E_2 follows the Negative-Binomial model $\text{Binomial}(r, \theta)$ with the target for number of successful trials $r > 0$ and the unknown probability of success parameter $\theta \in [0, 1]$.

Let H be the observed index to the experiment that is actually conducted, and let X denote the observed outcome of the conducted experiment.

Find a minimal sufficient statistic for θ and prove your claim.